Radiative seesaw in $SO(10)$ with dark matter

Mina K. Parida
Harish-Chandra Research Institute, Chhatnag Road, Jhusi, Allahabad 211019, India.
Email: paridam@mri.ernet.in

Abstract

High energy accelerators may probe into the dark matter and the seesaw neutrino mass scales if they are not much heavier than $\sim$O (TeV). In the absence of supersymmetry, we extend a class of minimal $SO(10)$ models to predict well known cold dark matter candidates while achieving precision unification with experimentally testable proton lifetime. The most important prediction is a new radiative seesaw formula of Ma type accessible to accelerator tests while the essential small value of its quartic coupling also emerges naturally. This dominates over the high-scale seesaw contributions making a major impact on neutrino physics and dark matter, opening up high prospects as a theory of fermion masses.

Introduction. Over the recent years, there has been a continued surge of interests in exploring the origin of dark matter of the universe while global efforts for understanding very small masses and large mixings in the neutrino sector have been intensified. The discovery of dark matter (DM) dates back to 1933 when, from velocity measurements in the Coma cluster, Zwicky predicted the inevitable presence of large clumps of massive nonluminous matter [1] which has been reconfirmed by a number of astrophysical and cosmological observations including the WMAP [2]. Based upon the gauge group $SU(2)_L \times U(1)_Y \times SU(3)_C (\equiv G_{213})$ the standard model (SM) predicts all neutrinos to be massless and no DM candidate. More than 70 years ago, Majorana conjectured neutrinos to be their own anti-particles and a neutrino mass may signify its Majorana character uncovering the violation of well known symmetry called the lepton number [3]. The revelation of tiny neutrino masses, intimately related to the neutrino oscillation phenomena which was at first hinted through Davis' Cl-37 experiment [4] in 1964, has been ultimately confirmed by atmospheric, solar, and reactor neutrino experiments [5]. Nearly four decades ago non-supersymmetric (non-SUSY) grand unified theories (GUTs) were proposed to unify three basic forces of nature with neat and robust prediction for proton decay, $p \to e^+\pi^0$, for which there are ongoing search experiments [6, 7]. Out of all GUTs, $SO(10)$ has grown in popularity as it can predict the right order of tiny neutrino masses through a path breaking new mechanism, called the canonical ($\equiv$ type-I) seesaw mechanism, shown to be possible only if neutrinos are Majorana fermions [8]. To mention a few out of a number of other qualities, while the model can explain the origins of parity (P) and CP violations, it has the potential for fitting all fermion masses and also explain baryon asymmetry of the universe via lepton asymmetry and sphaleron effects [10]. While the heavy right-handed (RH) neutrinos in $SO(10)$ mediate the canonical seesaw, the same theory also predicts another seesaw formula ($\equiv$ type-II) [9] but now mediated by a massive left-handed (LH) Higgs scalar triplet, Δ_L, and

the two mass formulas are

$$M_\nu^I = -M_D M_R^{-1} M_D^T, \quad M_\nu^{II} = f v_L, \tag{1}$$

where $v_L = \lambda V_R v^2/M_{\Delta_L}^2$, v = standard Higgs vaccum expectation value (VEV), V_R = VEV of RH Higgs triplet Δ_R, $M_R = f V_R$ = right-handed (RH) neutrino mass, M_D = Dirac mass of neutrino , and M_{Δ_L} = mass of Δ_L.

Available data on light neutrino masses constrain these scales to be high, $10^{13} - 10^{15}$ GeV and, as such, large hadron collider (LHC) and future high energy accelerators can not test the underlying origin of neutrino masses. Further, the minimal non-SUSY $SO(10)$ fails to fulfil the very purpose of unifying the SM gauge couplings for which it was designed, nor can it explain the dark matter phenomena.[1] However, supersymmetric (SUSY) $SO(10)$ with inbuilt Fermi-Bose symmetry achieves almost precision unification [11] and predicts dark matter with potential for TeV scale seesaw mechanism [12]. But SUSY GUTs have their own shortcomings too [13]. In any case, in the absence of any evidence of SUSY so far, it is worthwhile to explore prospects of non-SUSY SO(10) while preserving precision gauge coupling unification and dark matter as the twin guiding principles.

In this Letter we show how the minimal non-SUSY $SO(10)$ model is extended to predict DM and achieve precision unification with testable proton stability. With matter parity conservation, while type-I and type-II seesaw are automatic consequences of the model, it also generates a low-scale radiative seesaw formula of Ma type [14] accessible to accelerator tests and this formula dominates over the conventional ones causing a major impact on neutrino physics and dark matter phenomenology opening up high prospects as a theory of fermion masses in general. Comparison of prototon lifetime predictions of the present $SO(10)$ model and the $S(5) \times Z_2$ model [28] with the current experimental limit [7] for $p \to e^+\pi^0$ reveals clear distinction between the two models.

Precision unification. Prospective DM candidates are usually accommodated in model extensions by imposing additional discrete symmetries for their stability. But an encouraging aspect of non-SUSY $SO(10)$ is that when its gauged $U(1)_{B-L}$ subgroup breaks spontaneously by the same mechanism as the canonical seeesaw through the high scale VEV of the right-handed Higgs triplet, Δ_R, carrying $(B-L) = -2$, the surviving matter parity, $P_M = (-1)^{3(B-L)}$, emerging as gauged discreet symmetry Z_2 [15] can safeguard the stability of DM candiadtes once the latter are introduced into the model Lagrangian. The $SO(10)$ representations $10, 45, 54, 120, 126,$ and 210 possess even matter parity, but the representations $16, 144, ...$ have odd matter parity, irrespective of whether they represent fermions or scalars. Consistently, the SM fermions (Higgs) carry odd (even) matter parity. Therefore, the general principle for prospective DM particles is that, subject to fulfilment of all other phenomenological constraints, they might be nonstandard fermions of even P_M or scalars of odd P_M. Using suitable extensions of minimal non-SUSY $SO(10)$ while the hyperchargeless weak triplet fermion with well investigated phenomenology [16] has been predicted [18, 19], independently, the inert scalar doublet has also emerged as a CDM candidate [17]. But as we find here, both

[1] The well known minimal $SO(10)$ model is defined to be the one with standard fermion representation and Higgs representations necessary to implement the desert type spontaneous breaking and seesaw mechanisms.

the inert doublet and the fermion triplet can be made light, in addition to other non-standard fermions of $SO(10)$ leading to a substantial impact on neutrino physics, DM phenomenology, proton decay, and fermion masses.

For precision unification, at first we take out the scalar superpartners of quarks and leptons from the well known spectrum of the minimal supersymmetric standard model (MSSM). Then the remaining non-standard degrees of freedom due to the Higgs (χ) and fermions (F_i) in the non-SUSY model at low scale are [2]

$$\chi(2,1/2,1), F_\phi(2,1/2,1), F_\chi(2,-1/2,1), F_\sigma(3,0,1), F_b(1,0,1), F_C(1,0,8). \tag{2}$$

In eq.(2) F_ϕ, F_χ are analogues of two Higgsino doublets, F_σ, F_b and F_C are the analogues of wino, bino and gluino, and all the fermions except the octet have been treated as potential CDM candidates in SUSY GUTs. It is well known that without scalar superpartners, the fields in eq.(2) maitain unification of gauge coupling almost at the same scale and with the same level of precision as the MSSM but with a decreased value of the unification coupling. We note that when any one of the fields in eq.(2) with nontrivial quantum numbers is treated to be absent or made superheavy, the accuracy of precision unification at that scale is more or less reduced as in [17] while different combinations of fermions and scalars, but with exactly equivalent degrees of freedom as in eq.(2), yield unification with the same precision as in the MSSM as shown in ref. [19][3].

Using the SM particle masses and $m_{F_\phi} \simeq m_{F_\chi} = 2$ TeV, $m_{F_\sigma} \simeq m_\chi \simeq 3$ TeV, $m_{F_C} \simeq 6$ TeV, the resulting precision unification of gauge couplings in the non-SUSY theory occurs close to the MSSM GUT scale with $\mathrm{M}_U = 10^{15.96}$ GeV , $\alpha_G^{-1} = 35.3$. The closeness of the three couplings at the GUT scale is impressive, with $\alpha_1^{-1}(M_U) = 35.34, \alpha_2^{-1}(M_U) = 35.32$, and $\alpha_3^{-1}(M_U) = 35.30$ where $\alpha_i = g_i^2/(4\pi)$. The precision unification is guaranted in the presence of SM gauge symmetry below the GUT scale by assuming the superheavy Higgs components in each SO(10) representation to be degenerate in masses not very different from the GUT scale which reduce the GUT-threshold effects considerably [21, 22].

Like the SM fields, if the light fields given in eq.(2) can also be shown to emerge from suitable $SO(10)$ representations, then the non-SUSY GUT would be said to have realized the low-scale spectrum and this precision unification. For this purpose we exploit the non-standard fermionic representations, $45_F(+)$ and $10_F(+)$, in addition to the Higgs representations $10_H(+), \overline{16}_H(-), 45_H(+), \overline{126}_H(+), 54_H(+)$, and $210_H(+)$ where the respective matter parity, $(+)$ or $(-)$, has been shown against each representation. While the three fermions $F_\sigma, F_b, F_C \subset 45_F$, the fermion-doublet pair, $F_\phi, F_\chi \subset 10_F$. In order to make these nonstandard fermions light, we utilize the GUT-scale Yukawa Lagrangian

$$-L_{\rm Yuk} = 45_F\left(m_{45_F} + \lambda_P 210_H + \lambda_E 54_H\right)45_F + 10_F\left(m_{10_F} + \lambda_P' 45_H + \lambda_E' 54_H\right)10_F, \tag{3}$$

where generation indices have been suppressed. Utilizing the SM singlet VEVs in 54_H, 45_H, and 210_H, we find that the model has enough parameter space to make the four non-singlet fermions

[2] This may be recognized as the well known low scale spectrum in the split-SUSY model [20] except for the presence of an additional Higgs doublet.

[3] Unification of couplings with radiative seesaw and triplet DM has been also discussed outside $SO(10)$ with assumed discrete symmetry [28].

of eq.(2) and two singlet fermions F_b^i $(i=1,2)$ of the second and the third generations light by suitable tuning of the parameters in eq.(3) [24] while safeguarding precision unification in the same fashion as shown in ref.[19]. We will show that F_σ and F_b^i $(i=1,2)$ effectively replace the roles of RH neutrinos in driving the radiative seesaw. Similarly the non-standard inert doublet $\chi(2,1/2,1) \subset \overline{16}_H$ is brought to the $\sim$O(TeV) scale [25]. The presence of light fermions at low scales may be natural in non-SUSY GUTs as their masses could be protected by corresponding global symmetries. Perturbative and non-perturbative resolutions of cosmological relic density problem that might otherwise arise due to TeV scale mass of color octet fermion have been discussed earlier [19, 23].

To examine the impact of conserved matter parity on neutrino mass formulas we note that while the canonical and the type-II seesaw are automatic consequeces of this model, a number of other types of formulas normally allowed in the SM extensions or different SO(10) models [12] are now disallowed since, in the following Yukawa interaction,

$$-L'_{\rm Yuk} = Y 16_F 45_F \overline{16}_H, \tag{4}$$

the matter-parity violating VEV of $\overline{16}_H$ is forbidden. Further, matter-parity conserving type-I and type-II seesaw continue to remain as the only two formulas if, in eq.(2), the second Higgs doublet $\chi \subset 10_H(+)$ and carries even matter parity.

Radiative seesaw. The complexion of neutrino mass changes drastically once the second Higgs doublet χ in eq.(2) originates from $\overline{16}_H(-)$, carries odd matter parity, and aquires the status of an inert doublet [26, 27]. In addition to the Yukawa interaction in eq.(4), the following part of $SO(10)$-invariant Higgs potential is responsible for the radiative seesaw

$$\begin{aligned} V^U_{Higgs} &= m_{10}^2 10_H^2 + m_{16}^2 \overline{16}_H 16_H + \lambda_{10} 10_H^4 + \lambda_{16}(\overline{16}_H 16_H)^2 + \lambda_{\rm m}\overline{16}_H 16_H 10_H 10_H \\ &\quad +(\lambda_{\rm g}/{\rm M_{Pl}})\overline{16}_H 10_H.\overline{16}_H 10_H.\overline{126}_H. \end{aligned} \tag{5}$$

This leads to the low-scale Higgs potential

$$\begin{aligned} V &= m_\phi^2 \phi^\dagger\phi + m_\chi^2\chi^\dagger\chi + \frac{1}{2}\lambda_\phi(\phi^\dagger\phi)^2 + \frac{1}{2}\lambda_\chi(\chi^\dagger\chi)^2 + \lambda_1(\phi^\dagger\phi)(\chi^\dagger\chi) + \lambda_2(\phi^\dagger\chi)(\chi^\dagger\phi) \\ &\quad + \frac{1}{2}\lambda_3[(\phi^\dagger\chi)^2 + H.c.], \qquad \lambda_3 = \lambda_{\rm g} < \Delta_R > /{\rm M_{Pl}}. \end{aligned} \tag{6}$$

where ${\rm M_{Pl}} = 1.2 \times 10^{19}$ GeV. In the presence of precision unification with the SM gauge symmetry below the GUT scale, allowed natural value of $< \Delta_R > \sim 10^{16}$ GeV. With $\lambda_{\rm g} \sim O(1)$, the embedding of the radiative seesaw mechanism in this $SO(10)$ model then leads to the desired value of the quartic coupling, $\lambda_3 \simeq 10^{-5} - 10^{-3}$, covering the assumed value in ref. [14]. The expression for $\lambda_3 \propto < \Delta_R >$ in eq. (6) also serves as an anchor to type-I and type-II seesaw formulas. Thus, with the replacement of the externally imposed discrete symmetry of ref. [14] by the intrinsic matter parity ($\rm P_M$) and with the replacement of RH neutrinos of ref.[14] by adjoint fermions of this model, $(N_1, N_2, N_3) \to (F_\sigma, F_b^1, F_b^2)$, the radiatve seesaw mechanism emerges naturally.

Denoting ${\rm M}_{\chi_R}$ (${\rm M}_{\chi_I}$) as the mass of the real (imaginary) part of χ^0, it turns out that ${\rm M}^2_{\chi_R} - {\rm M}^2_{\chi_I} = 2\lambda_3 v^2$ while the charged component mass is ${\rm M}^2_{\chi^\pm} = {\rm M}^2_\chi + \lambda_1 v^2$. Under the assumption

that $\mathrm{M}^2_{\chi R} - \mathrm{M}^2_{\chi I} \ll \mathrm{M}^2_0 = (\mathrm{M}^2_{\chi R} + \mathrm{M}^2_{\chi I})/2$, which is easily satisfied because of the model prediction on the smallness of λ_3, the loop mediated radiative contribution is the same as in the derivation of Ma [14]

$$(\mathrm{M}^{\rm rad}_\nu)_{\alpha\beta} \;\; = \;\; \frac{\lambda_3 v^2}{8\pi^2}\mathbf{\Sigma}_i \frac{y_{\alpha i} y_{\beta i} F(\mathrm{M}^2_i/\mathrm{M}^2_0)}{\mathrm{M}^2_i}, \tag{7}$$

where $F(x) = [\lambda_3 v^2/(8\pi^2)][x/(1-x)]\,[1 + x\ln x/(1-x)]$. The formula in eq.(7) has been noted to give the resulting seesaw formulas in three limiting cases

$$\mathrm{M}^{\rm rad}_\nu \;\; = \;\; \frac{\lambda_3}{8\pi^2}\left[m_a \frac{1}{\mathrm{M}} m^T_a, \;\; \frac{m_a}{\mathrm{M}_0}\mathrm{M}(\frac{m_a}{\mathrm{M}_0})^T, \;\; m_a \frac{1}{\Lambda} m^T_a\right], \tag{8}$$

where the first, second, and the third entries hold for $\mathrm{M}^2_i \simeq \mathrm{M}^2_0$, $\mathrm{M}^2_i \ll \mathrm{M}^2_0$, and $\mathrm{M}^2_i \gg \mathrm{M}^2_0$, respectively, and we have defined $m_a = yv$, $\mathrm{M} = \mathrm{diag}(\mathrm{M}_1, \mathrm{M}_2, \mathrm{M}_3)$, $\Lambda_j = \mathrm{M}_j[\ln(\mathrm{M}^2_j/\mathrm{M}^2_0) - 1]^{-1}$, and $\Lambda = \mathrm{diag}(\Lambda_1, \Lambda_2, \Lambda_3)$. In general the neutrino mass matrix has a richer structure in this model due to tree-level and radiative seesaw contributions

$$\mathrm{M}_\nu = \mathrm{M}^I_\nu + \mathrm{M}^{II}_\nu + \mathrm{M}^{\rm rad}_\nu, \tag{9}$$

where the three terms on the RHS are given by eq.(1) and eq.(8).

Comparison and dominance. The most natural value of $\overline{126}_H$-Yukawa coupling to fermions is expected to be $f \simeq 1$ which imparts substantial contribution also to charged fermion masses near the GUT-scale [31, 32] derived from their low enegy values by renormalization group evolution [30]. In the present precision unification model, using $\mathrm{V}_R \simeq \mathrm{M}_{\Delta_L} \simeq \mathrm{M}_U \simeq 10^{16}$ GeV in eq.(1), we have for the third light neutrino mass, $m^I_3 \ll m^{II}_3 \sim 10^{-3}\lambda$ eV, which is at least one order smaller than the experimental value. Although the type-I contribution with fine-tuned value, $f \sim 0.01$, can yield the right order of neutino masses, its contribution to charged fermion masses is substantially weakened. There are SUSY $SO(10)$ models where type-II seesaw dominance has been shown to fit the charged fermion and neutrino sectors reasonably well [32]. In the present non-SUSY model the radiative seesaw can completely dominate and fit the available neutrino oscillation data. Compared to conventional $SO(10)$ models, the tension on f and other Yukawa couplings caused due to fitting the Dirac-neutrino masses and large neutrino mixings is absent in the present model. As a result, the Yukawa couplings of Higgs representations $\overline{126}_H$, 10_H, and 120_H get almost decoupled from the neutrino sector with their full potential to parametrize the charged fermion masses and mixings in a much more effective manner. This is natural as the radiative seesaw is basically designed to be more dominant as it admits much lighter seesaw scale. While details of these and a number of new $SO(10)$ applications [29] will be reported elsewhere, we confine here to the triplet fermion DM discussed earlier using SM extensions [16, 28].

Depending upon their actual masses, the Yukawa interaction in eq.(4) introduces decays, $\chi \to F_\sigma F^i_b$, $F_\sigma \to l\bar{l}F^i_b$, or $F^i_b \to l\bar{l}F_\sigma$ for $i = 1, 2$. Then only the lightest of them can be a stable dark matter candidate. Thus the model offers the possibility of fermionic weak triplet, singlet, or inert scalar doublet as a CDM candidate. In eq.(1) the Dirac neutrino mass, being of the same order as the up quark mass, the experimental value of large top-quark mass pushes the canonical

seesaw scale closer to the GUT scale. As there is no such constraint on the Yukawa couplings in eq.(4) and eq.(8), especially from experimental data, they can be small as has been assumed in [14]. Since the model permits additional lepton flavor violating processes compared to conventional SO(10) models, these couplings are constrained by $\mu \to e\gamma$ and other decay rates. For example, in order to have the triplet fermionic DM, the two adjoint singlet fermions and the inert scalar doublet are needed to be heavier than the triplet fermion and we examine this possibility including the new T2K data [33]. Denoting $\mathrm{M}_1 = m_{F_\sigma}, \mathrm{M}_2 = m_{F_b^1}$, and $\mathrm{M}_3 = m_{F_b^2}$, we choose $\mathrm{M}_1 < \mathrm{M}_2 < \mathrm{M}_3$ with $\mathrm{M_i}^2 \ll m_\chi^2$ for which the second relation of eq.(8) applies. Using neutrino mixing angles $\theta_{12} = 33^o, \theta_{23} = 43^o$, $\theta_{13} = 10^o$, and all phases to be vanishing, we have the mixing matrix elements $\mathrm{U}_{ei} = (0.808, 0.555, 0.190), \mathrm{U}_{\mu i} = (-0.661, 0.530, 0.666)$, and $\mathrm{U}_{\tau i} = (0.269, -0.638, 0.719)$. Using the relation $y_{\alpha i} = \mathrm{U}_{\alpha i} y_i$, the $\mu \to e\gamma$ decay rate constraint becomes $|(|y_1|^2 - (2/3)|y_2|^2 - (2/7)|y_3|^2)| \leq 0.672\ (m_\chi/2.7\ \mathrm{TeV})^2$ which is different from the tribimaximal mixing constraint [28]. With $\lambda_3 \sim O(10^{-5})$ and $y_i \sim O(10^{-2})$, the desired neutrino mass eigen values , $m_i = (0.010, 0.0135, 0.050)$ eV, in the hierarchial case are obtained for $\mathrm{M}_i = (2.7, 3.0, 3.3)$ TeV and $m_\chi = 40$ TeV . But we note that while all other parameters and predicted masses remain unchanged, the mass of the inert doublet is brought down to $m_\chi \sim 5$ TeV for $y_i \sim O(10^{-3})$. Inspite of nearly one order reduction in m_χ, the muon decay constraint is very well satisfied because of corresponding smaller values of the allowed Yukawa couplings. It is interesting to note that these couplings, $y_i \sim 10^{-2}(10^{-3})$, with adjoint fermions are of the same order as the charged-lepton Yukawa couplings for $\tau^-(\mu^-)$ in the non-SUSY SM. We also obtain solutions consistent with inverted hierarchy for $y_1 \leq y_2 \leq y_3$ in each case. The small values of y_i used here do not cause any problem since more rapid rate of annihilation and coannihilation required to produce right value of relic density of the triplet fermionic DM is accomplished by gauge boson interactions [16, 28]. We find that the masses of all the particles, mediating the radiative seesaw or responsible for its low scale, are in the range accessible to LHC or planned colliders.

Experimental signatures. It would be worthwhile to discuss some of the possible experimental signatures of this model which may distinguish it from the other non-SUSY GUT-based radiative seesaw model with $SU(5) \times Z_2$ grand unification symmetry [28].

(a)Proton lifetime predictions: $SO(10)$ vs. $SU(5)$. In order to make a possibly clear distinction we discuss gauge boson mediated proton decay $p \to e^+\pi^0$ for which there are ongoing dedicated experimental searches [7] with measured value of the lower limit on the life time, $\tau_p^{expt.} \geq 1.01 \times 10^{34}$ yrs. With a choice of TeV scale particle spectrum different from eq.(2), unification of couplings has been obtained in ref.[28] with $M_U^{SU(5)} = 2.65 \times 10^{15}$ GeV and the approximation adopted appears to predict proton lifetime substantially lower than the current experimental limit. Noting that the model prediction for the actual inverse decay rate has been underestimated, we re-evaluate it while estimating proton lifetime prediction in the present $SO(10)$ model. Including strong and electrowek renormalization effects on the d = 6 operator and taking into account quark mixing, chiral symmetry breaking effects, and lattice gauge theory estimations, the decay rates for the two models are [34, 35],

$$\Gamma(p \to e^+\pi^0) \quad = \quad \frac{m_p}{64\pi f_\pi^2} \frac{{g_G}^4}{{M_U}^4})|A_L|^2 |\bar{\alpha_H}|^2 (1 + D + F)^2 \times R,$$

$$\tag{10}$$

where $R = [A_{SR}^2 + A_{SL}^2(1+|V_{ud}|^2)^2]$ for $SU(5)$, but $R = [(A_{SR}^2 + A_{SL}^2)(1+|V_{ud}|^2)^2]$ for $SO(10)$, $V_{ud} = 0.974 =$ the $(1,1)$ element of V_{CKM} for quark mixings, and $A_{SL}(A_{SR})$ is the short-distance renormalization factor in the left (right) sectors. In eq.(10) $A_L = 1.25 =$ long distance renormalization factor which is the same for both models, but $A_{SL} \simeq A_{SR} = 2.414$ (2.542) for $SU(5)$ ($SO(10)$), $M_U =$ degenerate mass of 12 (24) superheavy gauge bosons in $SU(5)$ ($SO(10)$), $\bar{\alpha}_H =$ hadronic matrix elements, m_p =proton mass = 938.3 MeV, $f_\pi =$ pion decay constant = 139 MeV, and the chiral Lagrangian parameters are $D = 0.81$ and $F = 0.47$. With $\alpha_H = \bar{\alpha_H}(1 + D + F) = 0.012$ GeV^3 estimated from lattice gauge theory computations, we obtain $A_R \simeq A_L A_{SL} \simeq A_L A_{SR} \simeq 3.02$ (3.18) for $SU(5)$ ($SO(10)$), and the expression for the inverse decay rates for both the models is,

$$\Gamma^{-1}(p \to e^+\pi^0) = \frac{4}{\pi}\frac{f_\pi^2}{m_p}\frac{M_U^4}{\alpha_G^2}\frac{1}{\alpha_H^2 A_R^2}\frac{1}{F_q}, \tag{11}$$

where the GUT-fine structure constant $\alpha_G = 1/38.25$ and the factor $F_q = 1+(1+|V_{ud}|^2)^2 \simeq 4.8$ for $SU(5)$, but $\alpha_G = 1/35.3$ and $F_q = 2(1+|V_{ud}|^2)^2 \simeq 7.6$ for $SO(10)$. This formula reduces to the form given in [19, 35] and sets the lower limit for non-SUSY SU(5) GUT scale to be $M_U \geq 10^{15.5}$ GeV from the experimental lower limit on τ_p. Now using the estimated values of the model parameters in each case eq.(11) gives,

$$\begin{aligned} \tau_p^{SU(5)XZ_2} &\simeq 6.26 \times 10^{33} \ \ yrs, \\ \tau_p^{SO(10)} &\simeq 4.28 \times 10^{35} \ \ yrs. \end{aligned} \tag{12}$$

which mark clear distinction between the two models with much greater proton stability in the present model due to larger unification scale that originates from its TeV-scale spectrum. Thus, we have improved the proton lifetime estimation of [28] in the $SU(5) \times Z_2$ model by at least one order. The significance of this estimation is that the small deficit from the experimental lower limit can be compensated by invoking small threshold effects at the GUT-scale or the TeV scale. But there is a possibility that this model would be constrained if the future measurements increase the existing lower limit by about one order or longer.

In the present model, however, the one-loop prediction of proton lifetime is nearly 40 times longer than the current limit which may be accessed by the search experiments of the next generation[4]. The two-loop and small threshold effects [22] may also bring this prediction closer to the current experimental limit without disturbing precision unification. Since the TeV scale spectrum in the present model is richer and also two of the mediating fermions for radiative seesaw are different from the two RH neutrinos of ref.[28], there would be a variety of other possible ways by which this model can be experimentally distinguished from others. Pending these and other related investigations [36], the present estimation shows that improvement on

[4]As discussed in [19] the proton lifetime prediction in this model can also be reduced to nearly half of its predicted value when the scalar mediators in $\overline{10}_H \subset SU(5)$ contained in $\overline{16}_H \subset SO(10)$ have masses near $\sim$ TeV scale needed for shorter lifetime of cosmologically safe color-octet fermion [23].

the existing proton lifetime measurement by at least upto one order [7] would clearly favor the present $SO(10)$ based radiative seesaw model of precision unification.

(b)Accelerator tests. It has been shown that the color-octet fermion present in the TeV- scale spectrum would be pair produced at the LHC with nearly 1000, 15, and 2.5 number of events for its mass $m_{F_C} = 2.0$ TeV, 3.0 TeV, and 3.5 TeV, respectively, with 100fb^{-1} beam luminousity at energy $\sqrt{s} = 14$ TeV [19]. Its relic density problem can be evaded either non-perturbatively, or by invoking second inflation [23], or even perturbatively, by making its lifetime shorter [20] through the introduction of the complete Higgs multiplet $10_H \subset SU(5)$ contained in $16_H \subset SO(10)$ at an appropriate mass scale without disturbing precision unification. In the third case, there is a clear possibility of the boosted production of CDM candidates F_σ or F_b with displaced vertices at LHC via $F_C\bar{F}_C$ pair production [36]. Alternatively, the octet fermion can be replaced by a pair of color octet scalars to achieve the same precision unification and their LHC signatures have been discussed in detail [37]. They can be pair produced copiously at LHC energies and will manifest themselves as resonances in multijet final states. Another specific distinguishing signal at LHC or Tevatron, but more prominent at ILC, would be the pair production and decay of heavy charged fermions contained in F_ϕ and F_χ of the non-standard spectrum of the extended SO(10) model [36].

Summary and conclusion. While discrete symmetries are externally imposed on model extensions to maintain stability of incorporated dark matter, a minimal non-SUSY $SO(10)$ model naturally possess the stabilizing matter parity discrete symmetry, but it does not unify gauge couplings and neither does it predict prospective DM candidates. Although it predicts very attractive neutrino mass generation mechanisms, they involve high seesaw scales, $10^{13} - 10^{15}$ GeV, inaccessible for experimental tests in foreseeable future. Here the model is successfully extended to realize a low scale spectrum that achieves precision unification with experimentally testable proton lifetime, predicts an inert scalar doublet, and other potential fermionic DM candidates. With matter parity conservation, the type-I and type-II seesaw formulas are automatic consequences of the model, but mediated by the DM and the inert doublet, most interestingly, it also predicts the verifiable low-scale radiative seesaw formula of Ma type along with the natural emergence of its small quartic coupling. Moreover, the new contribution to neutrino mass dominates over the conventional ones making a major impact on neutrino physics and dark matter while opening up high potential as a theory of fermion masses in general. Our estimation in the $SU(5) \times Z_2$ based radiative seesaw model reveals the proton lifetime for $p \to e^+\pi^0$ to be somewhat less than the current experimental lower limit, but in the present $SO(10)$ model the lifetime turns out to be nearly 40 times longer which marks one of its clear distinguishing features.

Acknowledgment. M.K.P. thanks Harish-Chandra Research Institute for a visiting position.

References

[1] F. Zwicky, Helv. Phys. Acta **6** (1933) 110.

[2] D. N. Spergel *et al.*, WMAP Collaboration, Astrophys. J. Suppl. **170** (2007) 377; For a recent review, see for example, D. Hooper, TASI Lectures on Dark Matter, in *Boulder 2008, The dawn of the LHC era* (TASI 2008), pp 709-764.

[3] E. Majorana, Nuovo Cimento **14** (1937) 110.

[4] R. Davis, A, K. Mann, and L. Wolfenstein, Ann. Rev. Nucl. Part. Sci. **39** (1989) 467.

[5] For reviews see R. N. Mohapatra and A. Yu. Smirnov, Ann. Rev. Nucl. Part. Sci.**56** (2006) 569; G. Altarelli, arXiv:1107.1980 [hep-ph].

[6] J. C. Pati and A. Salam, Phys. Rev. Lett. **31** (1973) 661; Phys. Rev. **D 10** (1974) 275; H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32** (1974) 438; H. Georgi, in *Particles and Fields*, edited by C.E. Carlson (American Institute of Physics, New York, 1975); H. Fritzsch and P, Minkowski, *Ann. Phys. (N.Y.)* **93**, (1975) 193.

[7] H. Nishino *et al.*, Phys. Rev. Lett. **102** (2009) 141801; S. Raby *et al.*, arXiv:0810.4551 [hep-ph].

[8] P. Minkowski, Phys. lett. **B 67** (1977) 421; T. Yanagida, in *Proceedings of the Workshop on the Unified Theory and the Baryon Number in the Universe* (O. Sawada and A. Sugamoto, eds.), KEK, Tsukuba, Japan, 1979, p. 95; M. Gell-Mann, P. Ramond, and R. Slansky, *Supergravity* (P. van Nieuwenhuizen et al. eds.), North Holland, Amsterdam, 1980, p. 315; S. L. Glashow, *The future of elementary particle physics*, in *Proceedings of the 1979 Cargèse Summer Institute on Quarks and Leptons* (M. Lévy et al. eds.), Plenum Press, New York, 1980, pp. 687; R. N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44** (1980) 912.

[9] M. Magg and C. Wetterich, Phys. Lett. **B 94** (1980) 61; J. Schechter and J. W. F. Valle, Phys. Rev. **D 22** (1980) 2227; R. N. Mohapatra and G. Senjanovic, Phys. Rev. **D 23** (1981) 165; G. Lazaridis, Q. Shafi, and C. Wetterich, Nucl. Phys. **B 181** (1981) 287.

[10] M. Fukugita and T. Yanagida, Phys. Lett. **B 174** (1986) 45.

[11] W.J. Marciano and G. Senjanovic, Phys. Rev. **D 25** (1982) 3092; U. Amaldi, W. de Boer and H. Furstenau, Phys. Lett. **B 260** (1991) 447; P. Langacker and M. Luo, Phys. Rev. **D 44** (1991) 817.

[12] R. N. Mohapatra, Phys. Rev. Lett. **56** (1986) 561; R. Foot, H. Lew, X. G. He and G. C. Joshi, Z. Phys. **C 44** (1989) 441; E. Ma, Phys. Rev. Lett. **81** (2008) 1171; L. Boubekeur, T. Hambye and G. Senjanovic, Phys. Rev. Lett. **93**, (2004) 111601; S. M. Barr, Phys. Rev. Lett. **92** (2000) 101601; S. K. Majee, M. K. Parida and A. Raychaudhuri, Phys. Lett. **B 668** (2008) 299; M. K. Parida and A. Raychaudhuri, Phys. Rev. **D 82** (2010) 093017. For reviw see H. Hettamansperger, M. Lindner and W. Rodejohann, arXiv:1102.3432 [hep-ph].

[13] For finetunig problem and comparison of SUSY and non-SUSY theories see, for instance, S. M. Barr, Phys. Rev. **D 82** (2010) 055010; S. Weinberg, "Living in the Universe", in *Universe or Multiverse ?*, (B. J. Carr ed.), Cambridge Univ. Press, Cambridge, 2007, pp. 29.

[14] E. Ma, Phys. Rev. **D 73** (2006) 077301.

[15] R. N. Mohapatra, Phys. Rev. **D 34** (1986) 3457; L. M. Krauss and F. Wilczek, Phys. Rev. Lett. **62** (1989) 1221; S. P. Martin, Phys. Rev. **D 46** (1992) R2769; C. S. Aulakh, A. Melfo, A. Rasin, and G. Senjanovic, Phys. Rev. **D 358** (1998) 115007. For a review and other references see T. Hambye, arXiv:1012.4587 [hep-ph].

[16] M. Cirelli, N. Fornengo, and A. Strumia, Nucl. Phys. **B 753** (2006) 178. For a review see A. Strumia, in *Proceedings of Europhysics Conference on High Energy Physics*, HEP 2009, PoS EPS-HEP2009: 012 (2009).

[17] M. Kadastik, K. Kannike and M. Raidal, Phys. Rev. **D 80** (2009) 085020; *ibid.* Phys. Rev. **D 81** (2010) 015002.

[18] M. Frigerio and T. Hambye, Phys. Rev. **D 81** (2010) 075002.

[19] M. K. Parida, P. K. Sahu, and K. Bora, Phys. Rev. **D 83** (2011) 093004.

[20] N. Arkani-Hamed and S. Dimopoulos, JHEP **06** (2005) 073.

[21] V. V. Dixit and M. Sher, Phys. Rev. **D 40** (1989) 3765.

[22] M. K. Parida and P. K. Patra, Phys. Rev. Lett. **66** (1991) 858; ibid. **68** (1992) 74; R. N. Mohapatra, Phys. Lett. **B 285** (1992) 235; D.G. Lee, R. N. Mohapatra, M. K. Parida and M. Rani, Phys. Rev. **D 51** (1995) 229.

[23] H. Baer, K. Cheung, and J. F. Gunion, Phys. Rev. **D 59** (1999) 075002.

[24] T. Fukuyama, A. Ilakovac, S. Meljanac, and N. Okada, J. Math. Phys. **46** (2005) 033505; B. Bajc, A. Melfo, G. Senjanovic, and F. Vissani, Phys. Rev **D 70** (2004) 035007; C. S. Aulakh and A. Girdhar, Int. J. Mod. Phys. **A 20** (2005) 865.

[25] D. Chang, R. N. Mohapatra, and M. K. Parida, Phys. Rev. Lett. **52** (1984) 1072.

[26] N. G. Deshpande and E. Ma, Phys. Rev. **D 18** (1978) 2574.

[27] R. Barbieri, L. J. Hall and V. S. Rychkov, Phys. Rev. **D 74** (2006) 015007.

[28] E. Ma and D. Suematsu, Mod. Phys. Lett. **A 24** (2009) 583.

[29] E. Ma, Phys. Lett. **B 659** (2008) 885; J. Kubo, E. Ma and D. Suematsu, Phys. Lett. **B 642** (2006) 18; K. S. Babu and E. Ma, Int. J. Mod. Phys. **A 23** (2008) 1813; E. Ma, Phys. Rev. **D 80** (2009) 013013; J. Kubo and D. Suematsu, Phys. Lett. **B 643** (2006) 336; Pei-Hong Gu and U. Sarkar, Phys. Rev. **D 78** (2008) 073012; A. Adulpravitchai, M.

Lindner, A. Merle and R. N. Mohapatra, Phys. Lett. **B 680** (2009) 476; N. Haba and K. Tsumura, arxiv: 1105.1409 [hep-ph]; Y. Cai, X. G. He, M. Ramsey-Musolf and L. H. Tsai, arXiv:1108.0969 [hep-ph].

[30] C. R. Das and M. K. Parida, Eur. Phy. J. **C 20** (2001) 121.

[31] K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett.**70** (1993) 2845.

[32] See, for example, H. S. Goh, R. N. Mohapatra and S. Nasri, Phys. Rev. **D 70** (2004) 075022; B. Dutta, Y. Mimura and R.N. Mohapatra, Phys. Rev.**D 80** (2009) 095021; G.Altarelli and G. Blankenburg, JHEP **03** (2011) 133.

[33] K. Abe *et al.*, The T2K Collaboration, arXiv:1106.2822; X. G. He and A. Zee, arXiv:1106.4359 [hep-ph].

[34] B. Bajc, I. Dorsner and M. Nemevsek, JHEP **11** (2008) 007; P. Nath and P. F. Perez, Phys. Rep. **441** (2007) 191.

[35] K. S. Babu and J. C. Pati, JHEP **1006** (2010) 084.

[36] M. K. Parida, In preparation.

[37] B. A. Dobrescu, K. Kong and R. Mahbubani, JHEP **07** (2007) 006; Y. Bai and B. A. Dobrescu, JHEP **07** (2011) 100; S. Schumann, A. Renaud and D. Zerwas, arXiv:1108.2957.